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Effects of the stress field on the dynamic properties of masonry bell towers

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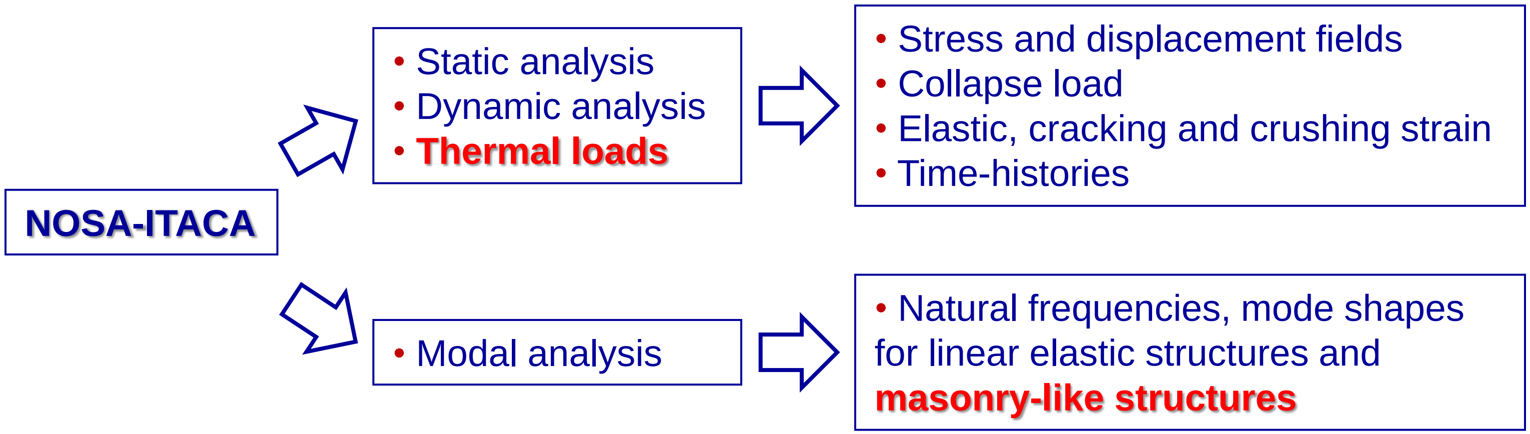
Summary :

1. The NOSA-ITACA software

- **NOSA-ITACA** is a **free** software package developed by ISTI-CNR. It is a finite element code that combines NOSA with the open source graphic platform SALOME. It has been specifically implemented to study the static and dynamic behavior of masonry constructions.

- **NOSA-ITACA** is distributed via the **http://www.nosaitaca.it/software/** website. The downloadable package includes SALOME v8.2.0, and is available for Ubuntu 14.04 and 16.04.

- **Masonry** is described as a nonlinear elastic material with zero or small tensile strength and infinite or bounded compressive strength.



1. The NOSA-ITACA software

The constitutive equation of **masonry-like** materials under non-isothermal conditions:

$\mathbf{E}$	infinitesimal strain tensor,
$\mathbf{T}$	Cauchy stress tensor,
$\mathbf{E}^e$	elastic part of the strain,
$\mathbf{E}^f$	fracture part of the strain,
$\mathbb{C}$	isotropic fourth-order tensor of elastic constants
$E,\ \nu$	Young 's modulus and Poisson's modulus,
α	linear coefficient of thermal expansion,
θ	absolute temperature,
θ_0	reference temperature

Given $\mathbf{E}$ and θ, it is possible to obtain $\mathbf{E}^f$, $\mathbf{E}^e$, $\mathbf{T}$ such that :

$$\mathbf{E} = \mathbf{E}^e + \mathbf{E}^f + \alpha(\theta - \theta_0)\,\mathbf{I}$$

$$\mathbf{T} = \mathbb{C}(E,\nu)[\mathbf{E}^e]$$

$$\mathbf{E}^f \cdot \mathbf{T} = 0$$

$$\mathbf{T} \in Sym^-,\ \mathbf{E}^f \in Sym^+$$

$\mathbf{T} = \check{\mathbf{T}}(\mathbf{E},\theta)$ stress function (explicit expression)

$\mathbf{D}_E\check{\mathbf{T}}(\mathbf{E},\theta)$ derivative of stress function with respect to the strain tensor (explicit expression)

2. Modal analysis of masonry structures: numerical method

The numerical procedure implemented in the NOSA–ITACA code allows to calculate the **natural frequencies and mode shapes of masonry buildings while taking into account the stress field due to mechanical and thermal loads as well.**

Standard modal analysis: constrained generalized eigenvalue problem

$$\boldsymbol{K}\phi = \omega^2 \boldsymbol{M}\,\phi \qquad \textbf{(1)}$$

$$\boldsymbol{T}\phi = 0 \qquad \textbf{(2)}$$

$\boldsymbol{K} \in \mathrm{R}^{nxn}$, stiffness matrix calculated assuming the structure's constituent material linear elastic; $\boldsymbol{M} \in \mathrm{R}^{nxn}$, mass matrix; n structure's total number of degrees of freedom; $\boldsymbol{T} \in \mathrm{R}^{mxn}$; m fixed constraints and the master–slave relations assigned to the displacements of the structure.

2. Modal analysis of masonry structures: numerical method

Case of masonry Structures

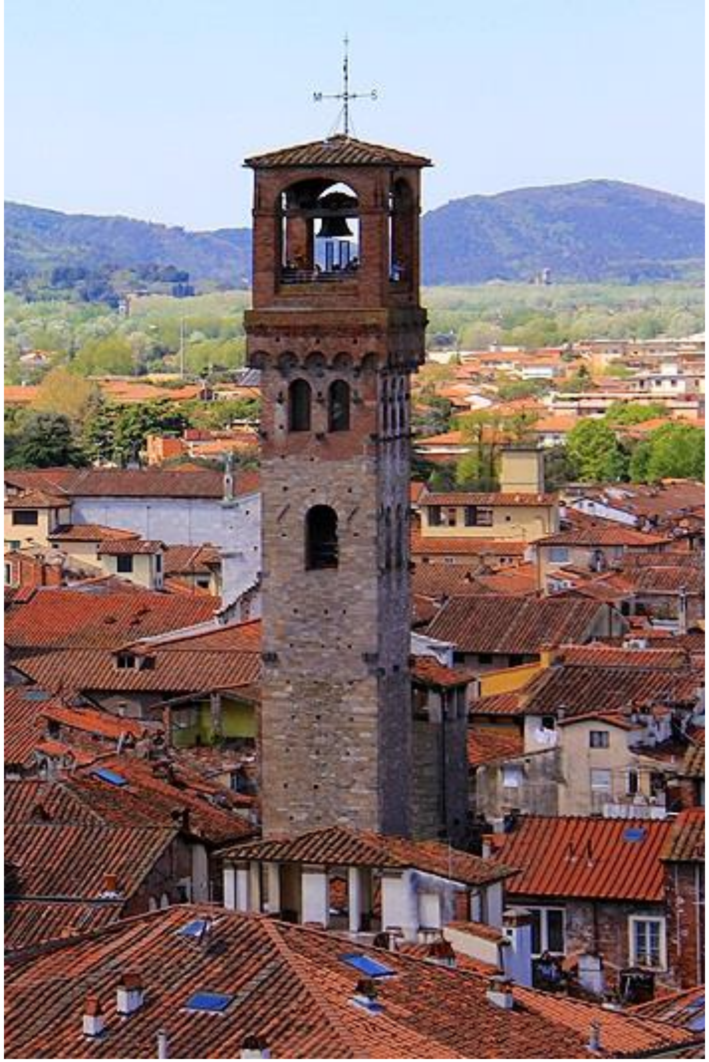
Step 1. The nonlinear equilibrium problem of the structure subjected to the assigned loads (mechanical and thermal) and boundary conditions is solved. The tangent stiffness matrix $\boldsymbol{K}^d$ is calculated by using the derivative of the stress with respect to the strain (explicit expression).

Step 2. The generalized eigenvalue problem (3)-(4), with matrix $\boldsymbol{K}^d$ in place of the elastic stiffness matrix $\boldsymbol{K}$, is solved, and the natural frequencies $f_i = \omega_i / 2\pi$ and mode shapes ϕ_i calculated.

$$\boldsymbol{K}^d \phi = \omega^2 \boldsymbol{M} \, \phi \qquad \textbf{(3)}$$

$$\boldsymbol{T} \phi = 0 \qquad \textbf{(4)}$$

3. The case study: the "Clock Tower" in Lucca

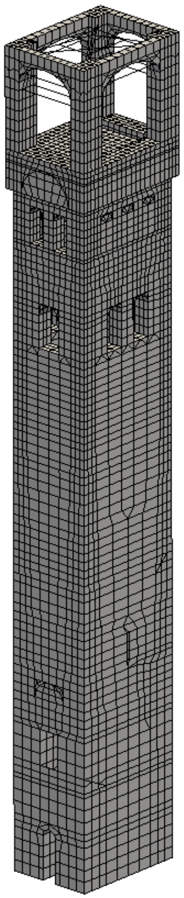


Characteristics of the tower

- Height 48.4m;
- Rectangular cross section of about 5.1x7.1m;
- Walls of variable thickness from about 1.77 m at the base to 0.85 m at the top;
- Two barrel vaults set at heights of about 12.5 and 42.3 m, respectively;
- Pavilion roof made up of wooden trusses and rafters;
- Adjacent buildings on two sides for a height of about 13 m (asymmetric boundary condition);
- Masonry of stone blocks and thin mortar joints and/or regular stone blocks and bricks, also with thin joints (no experimental information is available to date).

4. A finite element model of the tower

NOSA–ITACA code has been used, together with model updating techniques, in order to fit the experimental results obtained by an experimental campaign, in the case of a homogeneous **masonry-like** material.



Finite element model charateristics:

- 11383 brick elements;
- 34149 degrees of freedom;
- Structure clamped at the base and additional boundary conditions.

Frequencies: f_i^{exp} experimental, f_i numerical.

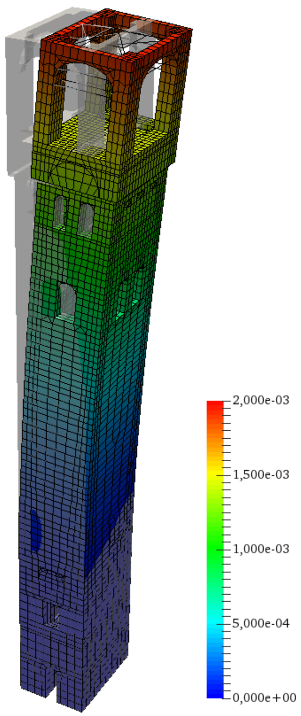
	f_i^{exp} [Hz]	f_i [Hz]	$\mid f_i^{exp}-f_i \mid / f_i^{exp}$ [%]
Mode shape 1	1.05	1.08	3.0
Mode shape 2	1.30	1.28	2.0
Mode shape 3	4.20	4.24	1.0
Mode shape 4	4.50	4.52	0.4

⇨ Masonry-like material:

E = 4.5 ·10^9 Pa
ρ = 2100 kg/m^3
ν =0.2

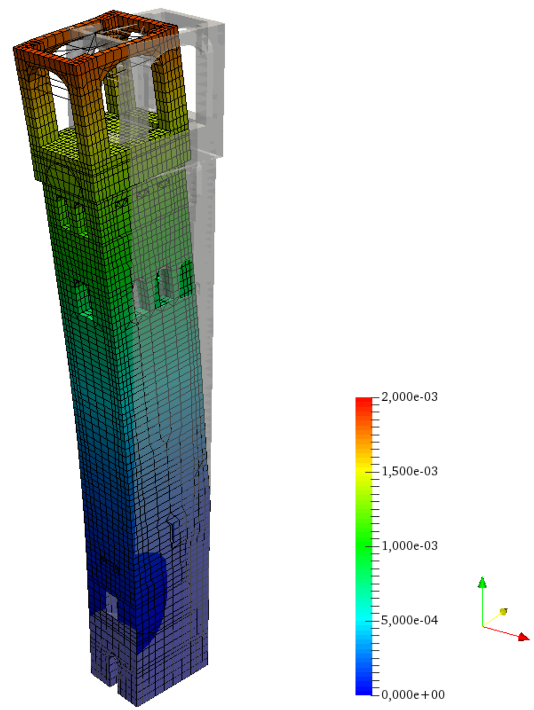
(optimal values)

4. A finite element model of the tower



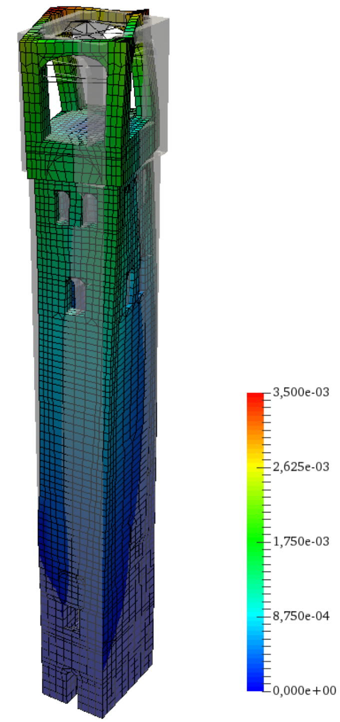
Mode 1
bending mode along X

f_1^{exp} = 1.05 Hz
f_1 = 1.08 Hz



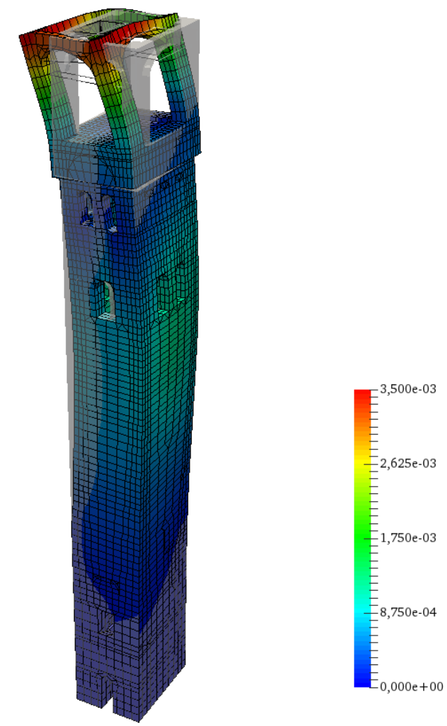
Mode 2
bending mode along Y

f_2^{exp} = 1.30 Hz
f_2 = 1.28 Hz



Mode 3
torsional

f_3^{exp} = 4.20 Hz
f_3 = 4.24 Hz

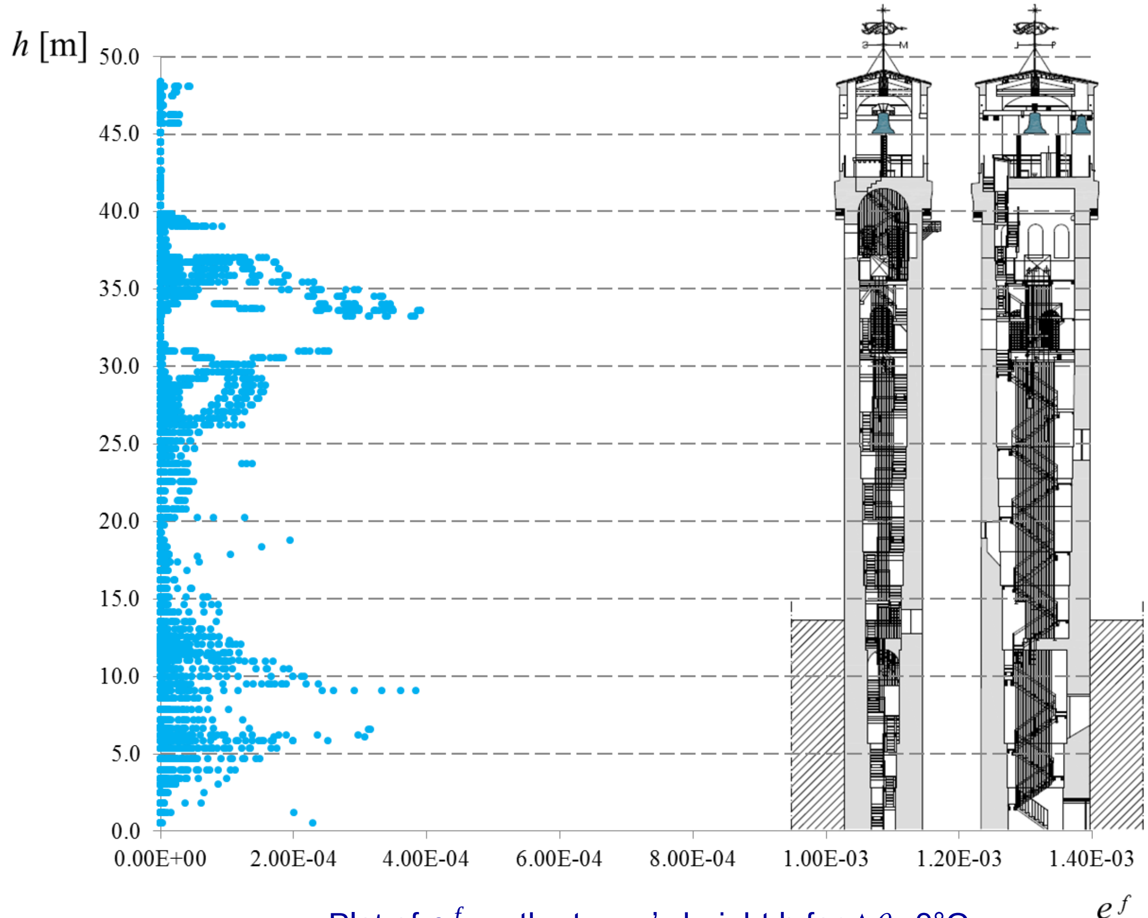


Mode 4
bending and torsional

f_4^{exp} = 4.50 Hz
f_4 = 4.52 Hz

5. Influence of temperature variations on the numerical frequencies

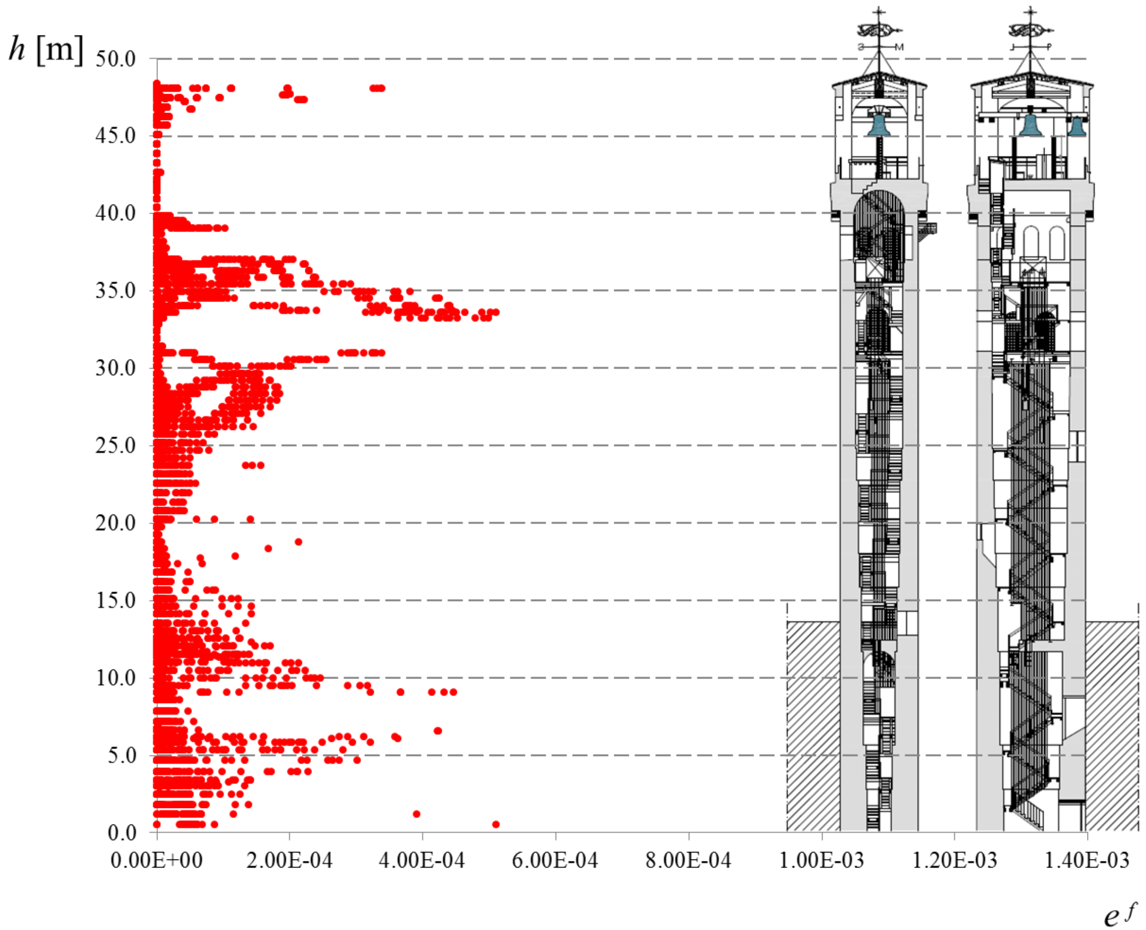
Application of the self weight and the uniform thermal load $\Delta\theta = \theta - \theta_0$ (mean yearly thermal variation): θ_0 = 10°C, θ_m = -5°C ($\Delta\theta$ = -15°C), θ_M = 30°C ($\Delta\theta$ = 20°C).



$e^f = (\int \mathrm{tr}\, \mathbf{E}^f\, dV)/V$

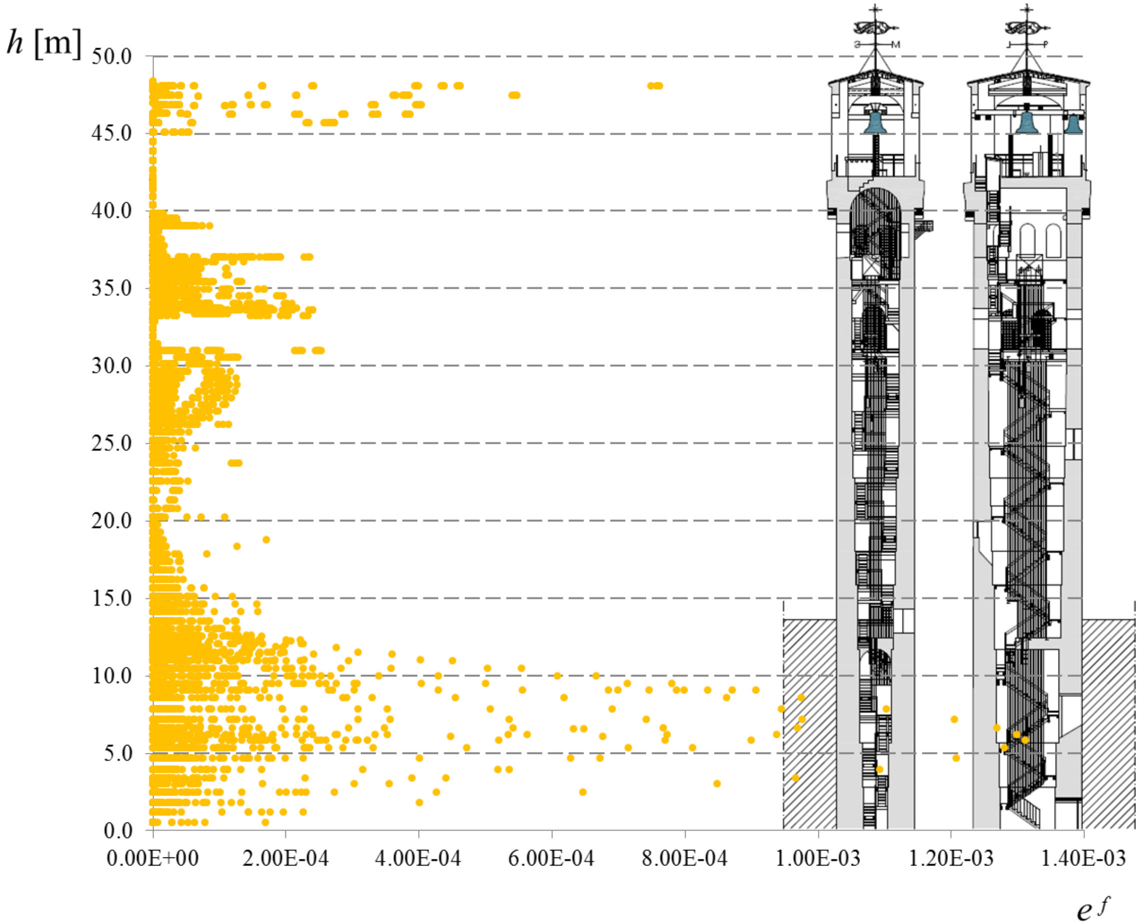
Plot of e^f vs. the tower's height h for $\Delta\theta$=0°C

5. Influence of temperature variations on the numerical frequencies



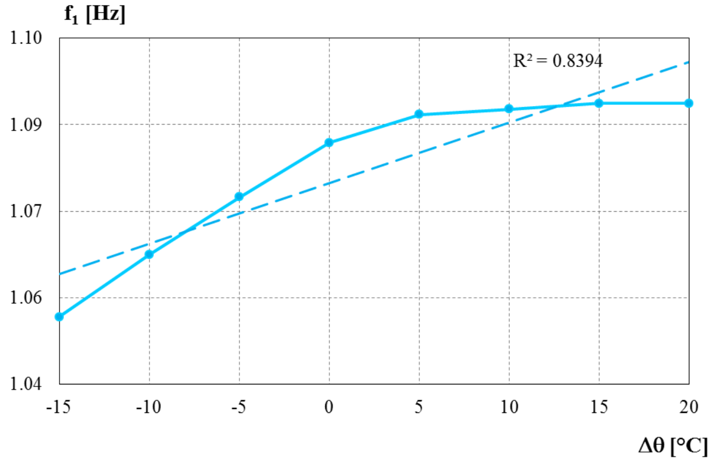
Plot of e^f vs. the tower's height h for $\Delta\theta$=-15°C

5. Influence of temperature variations on the numerical frequencies

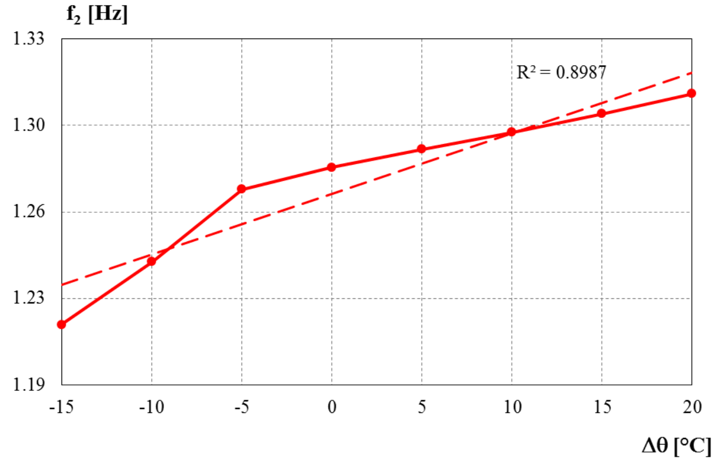


Plot of e^f vs. the tower's height h for $\Delta\theta$=20°C

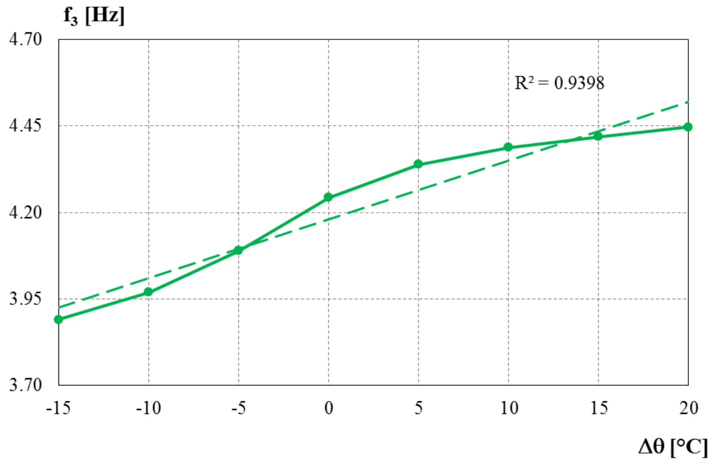
5. Influence of temperature variations on the numerical frequencies



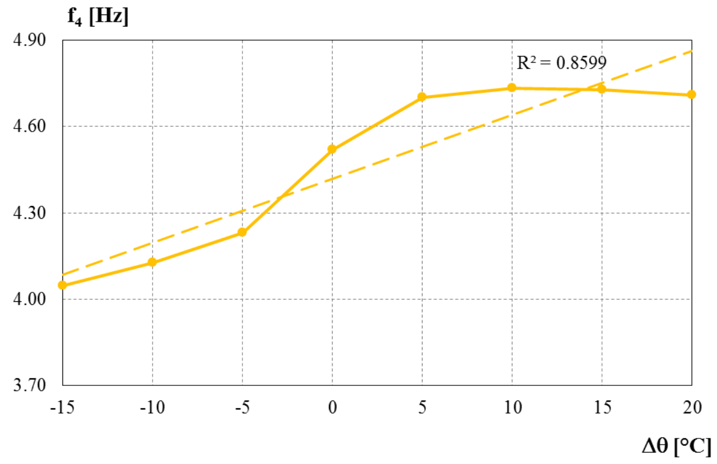
First natural frequency vs. $\Delta\theta$ and its regression line (dashed).



Second natural frequency vs. $\Delta\theta$ and its regression line (dashed).



Third natural frequency vs. $\Delta\theta$ and its regression line (dashed).



Fourth natural frequency vs. $\Delta\theta$ and its regression line (dashed).

5. Influence of temperature variations on the numerical frequencies

Frequencies variations

	f_i [Hz]	Δf_i^+ [%]	Δf_i^- [%]	R^2
Mode 1	1.08	0.67	2.97	0.84
Mode 2	1.28	2.33	4.98	0.90
Mode 3	4.24	4.79	8.32	0.94
Mode 4	4.52	4.19	10.48	0.86

Results

- In agreement with the experimental evidence the frequencies are increasing functions of θ.
- The frequencies are quite linearly correlated to temperature (R^2).

In order to compare the mode shapes ϕ_i and $\phi_j(\Delta\theta)$ we introduce the quantity:

$$\text{MAC-}M(\phi_i\,,\,\phi_j(\Delta\theta)\,) = |\phi_i\,\cdot \boldsymbol{M}\,\phi_j(\Delta\theta)\,|\,/\,((\phi_i\cdot\boldsymbol{M}\,\phi_i)^{1/2}(\phi_j(\Delta\theta)\,\cdot\boldsymbol{M}\,\phi_j(\Delta\theta))^{1/2}\,)$$

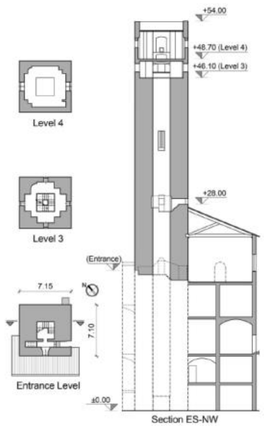
	ϕ_1	ϕ_2	ϕ_3	ϕ_4
$\phi_1(\Delta\theta_m)$	**0.99991**	0.00228	0.00253	0.00018
$\phi_2(\Delta\theta_m)$	0.00232	**0.99944**	0.01363	0.00920
$\phi_3(\Delta\theta_m)$	0.00268	0.01317	**0.95036**	0.28661
$\phi_4(\Delta\theta_m)$	0.00242	0.00460	0.27895	**0.94048**

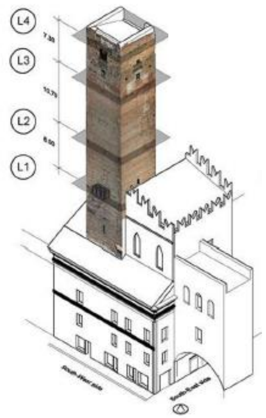
	ϕ_1	ϕ_2	ϕ_3	ϕ_4
$\phi_1(\Delta\theta_M)$	**0.99997**	0.00506	0.00039	0.00032
$\phi_2(\Delta\theta_M)$	0.00505	**0.99993**	0.00142	0.00007
$\phi_3(\Delta\theta_M)$	0.00039	0.00138	**0.99832**	0.02783
$\phi_4(\Delta\theta_M)$	0.00036	0.00013	0.02801	**0.99741**

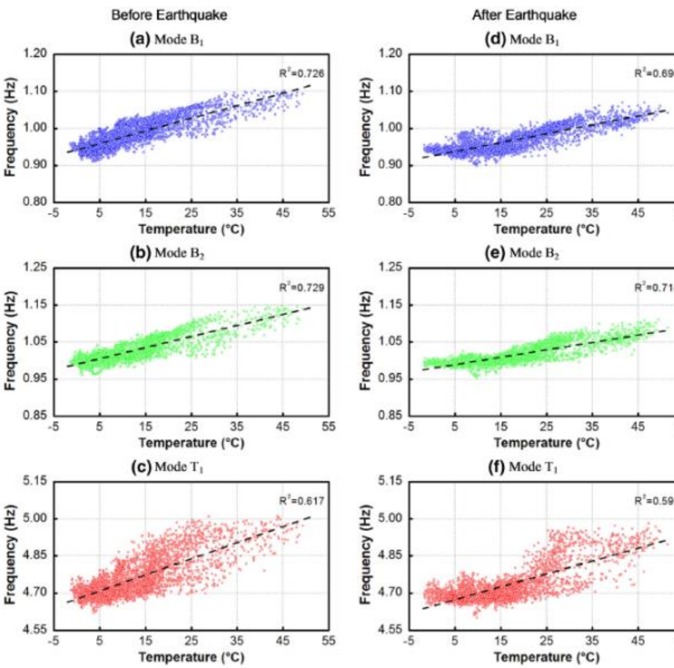
6. Influence of temperature variations on the experimental frequencies

Experimental results:

Gabbia tower, Mantova







C.Gentile, M. Guidobaldi, A. Saisi, *"One-year dynamic monitoring of a historic tower: damage detection under changing environment"*, Meccanica (2016) 51:2873–2889, DOI 10.1007/s11012-016-0482-3

7. Conclusion

- A new numerical procedure, implemented in the finite element code NOSA–ITACA, for the modal analysis of masonry structures is proposed (**http://www.nosaitaca.it/software/**).

- The procedure allows for taking into account the effects of the stress field due to thermal variations within the structure on its natural frequencies and mode shapes.

- The method proposed has been applied to the “Clock Tower” in Lucca subjected to its own weight and thermal loads simulating yearly temperature variations.

- The natural frequencies of the bell tower, calculated with **NOSA-ITACA**, are increasing functions of the temperature variation, in agreement with experimental data available in the literature.

Thank you

for your kind attention

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